On the higher-order deformed Heisenberg spin equation as an exactly solvable dynamical system

1989 J. Phys. A: Math. Gen. 22 4735

(http://iopscience.iop.org/0305-4470/22/21/034)

J. Phys. A: Math. Gen. 22 (1989) 4735–4736. Printed in the UK

COMMENT

On the higher-order deformed Heisenberg spin equation as an exactly solvable dynamical system

M Lakshmanan

Department of Physics, Bhrathidasan University, Tiruchirapalli-620 024, India

Received 29 June 1989

Abstract. It is pointed out that the contents of the recent article in this journal by De-gang Zhang and Jie Liu regarding the geometrical equivalence of a deformed continuous spin equation to the Hirota equation are merely a special case of a more generalised set-up already published in the literature some time ago.

In a recent letter in this journal, De-gang Zhang and Jie Liu (1989) have noted that the higher-order deformed Heisenberg spin equation

$$\begin{aligned} &\boldsymbol{S}_t = \boldsymbol{S} \wedge \boldsymbol{S}_{xx} - 3\alpha(\boldsymbol{S}_x \cdot \boldsymbol{S}_x)\boldsymbol{S}_x - 6\alpha(\boldsymbol{S}_x \cdot \boldsymbol{S}_{xx})\boldsymbol{S} - 2\alpha\boldsymbol{S}_{xxx} = 0 \\ &\boldsymbol{S}^2 = 1 \qquad \boldsymbol{S} = (S_1, S_2, S_3) \end{aligned} \tag{1}$$

is geometrically equivalent to the Hirota equation

$$\mathrm{i}\psi_t + \psi_{xx} + \tfrac{1}{2}|\psi|^2\psi + 3\mathrm{i}\alpha|\psi|^2\psi_x + 2\mathrm{i}\alpha\psi_{xxx} = 0. \tag{2}$$

It is the purpose of this comment to point out that this result is simply a special case of the more general equivalence shown by Lakshmanan and Ganesan (1983, 1985), where the generalised Heisenberg spin equation including linear inhomogeneities,

$$\begin{aligned} &\boldsymbol{S}_t = (\gamma_2 + \mu_2 x)\boldsymbol{S} \wedge \boldsymbol{S}_{xx} + \mu_2 \boldsymbol{S} \wedge \boldsymbol{S}_x - (\gamma_1 + \mu_1 x)\boldsymbol{S}_x - \gamma(\boldsymbol{S}_{xx} + \tfrac{3}{2}\boldsymbol{S}_x^2\boldsymbol{S})_x \\ &\boldsymbol{S}^2 = 1 \qquad \boldsymbol{S} = (S_1, S_2, S_3) \end{aligned} \tag{3}$$

is proved to be geometrically as well as gauge equivalent to the generalised non-linear Schrödinger equation with linear inhomogeneities,

$$\begin{aligned} &\mathrm{i}q_t + \mathrm{i}\mu_1 q + \mathrm{i}(\gamma_1 + \mu_1 x)q_x + (\gamma_2 + \mu_2 x)(q_{xx} + 2|q|^2 q) \\ &\quad + 2\mu_2\left(q_x + q\int_{-\infty}^{x} |q|^2\, \mathrm{d}x'\right) + \mathrm{i}\gamma(q_{xxx} + 6|q|^2 q_x) = 0. \end{aligned} \tag{4}$$

It is trivial to see that the systems (1) and (2) are simply particular cases of (3) and (4), respectively, for the parametric choice

$$\gamma_1 = \mu_1 = \mu_2 = 0 \qquad \gamma_2 = 1 \qquad \gamma = 2\alpha \tag{5}$$

along with a rescaling $q = \psi/2$.

The generalised equations (3) and (4) correspond to non-isospectral flows, in contrast to the special cases (1) and (2) and possess an infinite number of conservation laws, etc, but of more complicated type. The associated inverse scattering problems have also been discussed in Lakshmanan and Ganesan (1983, 1985).

References

De-gang Zhang and Jie Liu 1989 *J. Phys. A: Math. Gen.* **22** L53-4
Lakshmanan M and Ganesan S 1983 *J. Phys. Soc. Japan* **52** 4031-3
—— 1985 *Physica* **132A** 117-42